

Integration By Parts:

$$\int x \sin x dx = -x \cos x - \int -\cos x dx = -x \cos x + \int \cos x dx = -x \cos x + \sin x$$

1. $u = x \Rightarrow du = dx$

$$dv = \sin x dx \Rightarrow v = \int \sin x dx \Rightarrow v = -\cos x$$

2.

$$\int x^2 e^{2x} dx = x^2 \cdot \frac{1}{2} e^{2x} - \int \frac{1}{2} e^{2x} \cdot 2x dx = \frac{1}{2} x^2 e^{2x} - \int x e^{2x} dx$$

$$u = x^2 \Rightarrow du = 2x dx$$

$$dv = e^{2x} dx \Rightarrow v = \int e^{2x} dx = \frac{1}{2} \int e^z dz = \frac{1}{2} e^z = \frac{1}{2} e^{2x}$$

$$z = 2x \Rightarrow dz = 2 dx \Rightarrow \frac{1}{2} dz = dx$$

$$\frac{1}{2} x^2 e^{2x} - \int x e^{2x} dx = \frac{1}{2} x^2 e^{2x} - \left[x \cdot \frac{1}{2} e^{2x} - \int \frac{1}{2} e^{2x} dx \right] =$$

$$dv = e^{2x} dx \Rightarrow v = \int e^{2x} dx = \frac{1}{2} \int e^z dz = \frac{1}{2} e^z = \frac{1}{2} e^{2x}$$

$$\frac{1}{2} x^2 e^{2x} - x \cdot \frac{1}{2} e^{2x} + \frac{1}{2} \int e^{2x} dx = \frac{1}{2} x^2 e^{2x} - x \cdot \frac{1}{2} e^{2x} + \frac{1}{2} \cdot \frac{1}{2} e^{2x} = \frac{1}{2} x^2 e^{2x} - x \cdot \frac{1}{2} e^{2x} + \frac{1}{4} e^{2x} =$$
$$\frac{1}{4} e^{2x} (2x^2 - 2x + 1)$$

$$\int x^4 \ln x dx = \ln x \cdot \frac{x^5}{5} - \int \frac{x^5}{5} \cdot \frac{1}{x} dx = \frac{x^5 \ln x}{5} - \frac{1}{5} \int x^4 dx = \frac{x^5 \ln x}{5} - \frac{1}{5} \cdot \frac{x^5}{5} = \frac{x^5 \ln x}{5} - \frac{x^5}{25} =$$

3. $\frac{x^5}{25} (5 \ln x - 1)$

$$u = \ln x \Rightarrow du = \frac{1}{x} dx$$

$$dv = x^4 dx \Rightarrow v = \int x^4 dx = \frac{x^5}{5}$$

$$\int \sqrt{x} \ln(2x) dx = \int x^{\frac{1}{2}} \ln(2x) dx = \ln(2x) \cdot \frac{2}{3} x^{\frac{3}{2}} - \int \frac{2}{3} x^{\frac{3}{2}} \cdot \frac{1}{x} dx = \frac{2}{3} x^{\frac{3}{2}} \ln(2x) - \frac{2}{3} \int x^{\frac{1}{2}} dx =$$

$$\frac{2}{3} x^{\frac{3}{2}} \ln(2x) - \frac{2}{3} \cdot \frac{x^{\frac{3}{2}}}{\frac{3}{2}} = \frac{2}{3} x^{\frac{3}{2}} \ln(2x) - \frac{4}{9} x^{\frac{3}{2}} = \frac{2}{3} x^{\frac{3}{2}} \left(\ln(2x) - \frac{2}{3} \right)$$

4.

$$u = \ln(2x) \Rightarrow du = \frac{1}{2x} \cdot 2 dx \Rightarrow du = \frac{1}{x} dx$$

$$dv = x^{\frac{1}{2}} dx \Rightarrow v = \int x^{\frac{1}{2}} dx = \frac{x^{\frac{3}{2}}}{\frac{3}{2}} = \frac{2}{3} x^{\frac{3}{2}}$$

5.

$$\int x \sqrt{3-x} dx = \int x(3-x)^{\frac{1}{2}} dx = x \cdot -\frac{2}{3} (3-x)^{\frac{3}{2}} - \int -\frac{2}{3} (3-x)^{\frac{3}{2}} dx = -\frac{2}{3} x(3-x)^{\frac{3}{2}} + \frac{2}{3} \int (3-x)^{\frac{3}{2}} dx =$$

$$u = x \Rightarrow du = dx$$

$$dv = (3-x)^{\frac{1}{2}} dx \Rightarrow v = \int (3-x)^{\frac{1}{2}} dx = -\int z^{\frac{1}{2}} dz = -\frac{z^{\frac{3}{2}}}{\frac{3}{2}} = -\frac{2}{3} z^{\frac{3}{2}} = -\frac{2}{3} (3-x)^{\frac{3}{2}}$$

$$z = (3-x) \Rightarrow dz = -dx \Rightarrow -dz = dx$$

$$-\frac{2}{3} x(3-x)^{\frac{3}{2}} + \frac{2}{3} \cdot -1 \int u^{\frac{3}{2}} du = -\frac{2}{3} x(3-x)^{\frac{3}{2}} - \frac{2}{3} \cdot \frac{u^{\frac{5}{2}}}{\frac{5}{2}} = -\frac{2}{3} x(3-x)^{\frac{3}{2}} - \frac{4}{15} u^{\frac{5}{2}} = -\frac{2}{3} x(3-x)^{\frac{3}{2}} - \frac{4}{15} (3-x)^{\frac{5}{2}} =$$

$$u = (3-x) \Rightarrow du = -dx \Rightarrow -du = dx$$

$$-\frac{2}{3} (3-x)^{\frac{3}{2}} \left(x - \frac{2}{5} (3-x) \right)$$

6.

$$\int x^2 \sqrt{x+5} dx = \int x^2 (x+5)^{\frac{1}{2}} dx = x^2 \cdot \frac{2}{3} (x+5)^{\frac{3}{2}} - \int \frac{2}{3} (x+5)^{\frac{3}{2}} \cdot 2x dx = \frac{2}{3} x^2 (x+5)^{\frac{3}{2}} - \frac{4}{3} \int x (x+5)^{\frac{3}{2}} dx$$

$$u = x^2 \Rightarrow du = 2x dx$$

$$dv = (x+5)^{\frac{1}{2}} dx \Rightarrow v = \int (x+5)^{\frac{1}{2}} dx = \int z^{\frac{1}{2}} dz = \frac{z^{\frac{3}{2}}}{\frac{3}{2}} = \frac{2}{3} z^{\frac{3}{2}} = \frac{2}{3} (x+5)^{\frac{3}{2}}$$

$$z = (x+5) \Rightarrow dz = dx$$

$$\frac{2}{3} x^2 (x+5)^{\frac{3}{2}} - \frac{4}{3} \left[x \cdot \frac{2}{5} (x+5)^{\frac{5}{2}} - \int \frac{2}{5} (x+5)^{\frac{5}{2}} dx \right] = \frac{2}{3} x^2 (x+5)^{\frac{3}{2}} - \frac{4}{3} \left[x \cdot \frac{2}{5} (x+5)^{\frac{5}{2}} - \frac{2}{5} \int (x+5)^{\frac{5}{2}} dx \right] =$$

$$u = x \Rightarrow du = dx$$

$$dv = (x+5)^{\frac{3}{2}} dx \Rightarrow v = \int (x+5)^{\frac{3}{2}} dx = \int z^{\frac{3}{2}} dz = \frac{z^{\frac{5}{2}}}{\frac{5}{2}} = \frac{2}{5} z^{\frac{5}{2}} = \frac{2}{5} (x+5)^{\frac{5}{2}}$$

$$z = (x+5) \Rightarrow dz = dx$$

$$\frac{2}{3} x^2 (x+5)^{\frac{3}{2}} - \frac{4}{3} \left[x \cdot \frac{2}{5} (x+5)^{\frac{5}{2}} - \frac{2}{5} \int u^{\frac{5}{2}} dx \right] = \frac{2}{3} x^2 (x+5)^{\frac{3}{2}} - \frac{4}{3} \left[x \cdot \frac{2}{5} (x+5)^{\frac{5}{2}} - \frac{2}{5} \cdot \frac{u^{\frac{7}{2}}}{\frac{7}{2}} \right] =$$

$$u = (x+5) \Rightarrow du = dx$$

$$\frac{2}{3} x^2 (x+5)^{\frac{3}{2}} - \frac{4}{3} \left[x \cdot \frac{2}{5} (x+5)^{\frac{5}{2}} - \frac{4}{35} \cdot (x+5)^{\frac{7}{2}} \right] = \frac{2}{3} x^2 (x+5)^{\frac{3}{2}} - \frac{8}{15} x (x+5)^{\frac{5}{2}} + \frac{16}{105} (x+5)^{\frac{7}{2}} =$$

$$\frac{2}{3} (x+5)^{\frac{3}{2}} \left(x^2 - \frac{4}{5} x (x+5) + \frac{8}{35} (x+5)^2 \right)$$